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# WEATHER PREDICTION AND THE BUTTERFLY EFFECT

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***Abstract***

*This paper deals with a summary presentation of the butterfly effect and of some facts and experiments that makes this effect an important tool for the study of meteorology, especially for weather prediction.*

**Key words:** attractor**,** chaos theory, iterative system, Lorenz attractor, weather pattern

## INTRODUCTION

Weather prediction is an extremely difficult problem. Meteorologists can predict the weather for short periods of time, a couple days at most, but beyond that predictions are generally poor. Weather patterns are an example of iterative systems that can exhibit chaotic behaviour. In everyday language "chaos" implies the existence of unpredictable or random behaviour. The word usually carries a negative connotation involving undesirable disorganization or confusion. However, in the scientific realm this unpredictable behaviour is not necessarily undesirable. In short, chaos embodies three important principles:extreme sensitivity to initial conditions, cause and effect are not proportional and nonlinearity.

## WEATHER PREDICTION

The forecast models will not be very reliable at predicting the exact location of the events above. However, the models can give you insight into the potential of the event occurring. This is why weather forecasting is full of predictions that include a probability of occurrence. Weather features often occur on too small of a scale to be realistically forecasted over a particular location.
Probability forecasting basically eliminates this problem. For example, giving a probability of thunderstorms for a city is a more realistic prediction than a forecast saying there will or will not be a thunderstorm at a city. If a thunderstorm misses a city by a few miles when there was a probability of thunderstorms in the forecast it is not a busted forecast. However, if there is a high number probability of thunderstorms in the area and nothing happens in or near the forecast area then this would be a bust.
The smaller the scale of a weather feature the more difficulty the forecast models will have in resolving that feature. This is important because smaller scale features influence larger scale features more significantly as time passes. This fact is what causes the forecast models to tend to be less accurate as time moves forward. This is termed "the butterfly effect". Just a person breathing can have an influence on the weather over time. The smallest of events can have the largest of outcomes over time.
Weather patterns are an example of iterative systems that can exhibit chaotic behaviour. An *iterative system* is simply a function where the output for the next step, or *iteration*, is dependent on the result from the previous iteration. This sort of system can be applied to real life, where all sorts of things depend on their current state.
Edward Lorenz was a mathematician and meteorologist at the Massachusetts Institute of Technology . With the advent of computers, Lorenz saw the chance to combine mathematics and meteorology. He set out to construct a

mathematical model of the weather, namely a set of differential equations that represented changes in temperature, pressure, wind velocity, etc. In the end, Lorenz stripped the weather down to a crude model containing a set of 12 differential equations.

MOTION OF ATMOSPHERIC FLUID

Fluids can transfer heat. For fluids with high Reynolds numbers (chaotic fluids), the fraction of the kinetic energy of the fluid particles that is dissipated by friction as heat is small. Thus, the air, which we may call the atmospheric fluid is most adept at transporting heat ought to be turbulent. It is thanks to this turbulent air that we are alive today. Air's thermal conductivity is low, so when the Earth is heated by the Sun during the day, it does not emit much of this heat into the air via conduction. If winds (and, therefore, convective turbulence) did not exist, then the Earth would have a lot of excess thermal energy each day and would eventually burn us, as surface temperatures could easily reach 373 K!

Another example of how turbulence protects us pertains to carbon dioxide. Since carbon dioxide is the heaviest component of the atmosphere, if the air were motionless, then all of the carbon dioxide in the atmosphere would hover at very low altitudes, poisoning us all. However, there is turbulence, and there are winds, so the carbon dioxide is kept dispersed throughout the atmosphere.

THE WIND

Wind is indeed the flow of a fluid, namely the air, and is a key factor in weather conditions.

Winds are caused by the sum of internal and external forces on the air. Air experiences a gravitational force downward. Under calm conditions, the air is held roughly at the same altitude because the pressure gradient force counteracts the gravitational force.

Whenever the magnitude of the pressure gradient force varies significantly from that of the gravitational force, then a wind begins to blow. The air pressure varies with altitude such that

$$\frac{\partial p}{\partial z} = -\frac{gM}{RT}\,p\,, \qquad (1)$$

where $z$ is the altitude, $R$ is the universal gas constant, and $M$ is the molar mass of air in kilograms. Temperature gradients with respect to altitude tend to be rather small, so at relatively high altitudes, we can claim that the temperature is roughly constant over changes in altitude that are not very large. In that case, we can say that

$$p = p_0 e^{-gMz/RT}\,, \qquad (2)$$

where $p_0$ is the air pressure at the Earth's surface. Here the temperature is usually assumed to be about 250K. Under calm conditions, the resulting pressure gradient force should be roughly equal in magnitude and opposite in direction to the gravitational force.

Unbalanced pressure gradient forces in horizontal directions lead to wind development just as they do in the vertical direction.

THE BUTTERFLY EFFECT

On a particular day in the winter of 1961, Lorenz wanted to re-examine a sequence of data coming from his model. Instead of restarting the entire run, he decided to save time and restart the run from somewhere in the middle. Using data printouts, he entered the conditions at some point near the middle of the previous run, and re-started the model calculation. What he found was very unusual and unexpected. The data from the second run should have exactly matched the data from the first run. While they matched at first, the runs eventually began to diverge dramatically — the second run losing all resemblance to the first within a few "model" months. A sample of the data from his two runs is shown below:

Fig. 1.

Lorenz finally found the source of the problem. To save space, his printouts only showed three digits while the data in the computer's memory contained six digits. Lorenz had entered the rounded-off data from the printouts assuming that the difference was inconsequential. For example, even today temperature is not routinely measured within one part in a thousand.

This led Lorenz to realize that long-term weather forecasting was doomed. His simple model exhibits the phenomenon known as "sensitive dependence on initial conditions." This is sometimes referred to as the butterfly effect, e.g. a butterfly flapping its wings in South America can affect the weather in Central Park. The question then arises — why does a set of completely deterministic equations exhibit this behaviour? After all, scientists are often taught that small initial perturbations lead to small changes in behaviour. This was clearly not the case in Lorenz's model of the weather. The answer lies in the nature of the equations; they were

nonlinear equations. While they are difficult to solve, nonlinear systems are central to chaos theory and often exhibit fantastically complex and chaotic behaviour.

Edward Lorenz's first weather model exhibited chaotic behaviour, but it involved a set of 12 nonlinear differential equations. Lorenz decided to look for complex behaviour in an even simpler set of equations, and was led to the phenomenon of rolling fluid convection. The physical model is simple: place a gas in a solid rectangular box with a heat source on the bottom.

Lorenz simplified a few fluid dynamics equations - called the Navier-Stokes equations - and ended up with a set of three nonlinear equations:

$$dx/dt = P(y - x)$$

$$dy/dt = Rx - y - xz$$

$$dz/dt = xy - By \tag{3}$$

where $P$ is the Prandtl number representing the ratio of the fluid viscosity to its thermal conductivity, $R$ represents the difference in temperature between the top and bottom of the system, and $B$ is the ratio of the width to height of the box used to hold the system. The   simple to solve. However, they represent an extremely complicated dynamical system.

LORENZ ATTRACTOR

If one plots the results in three dimensions the following figure, called the Lorenz attractor, is obtained.

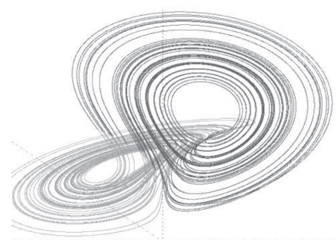

Fig. 2.

The projection on the y-z plane is shown below:

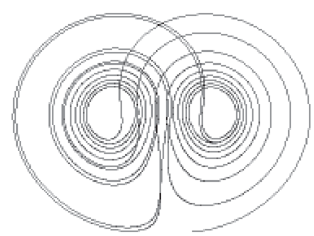

Fig. 3.

The projection on the x-z plane is also shown below:

Fig. 4.

## LORENZ EQUATIONS

If we vary all three parameters in the Lorenz equations, we will find many different types of solutions. For some sets of parameters, the solution will exhibit **preturbulence**, which is where trajectories oscillate chaotically for a while before reaching a stable stationary or periodic behaviour. Others yield **intermittent chaos**, which is where trajectories alternate between chaos and stable periodic behaviour. Still others lead to **noisy periodicity**, which is where trajectories are very close to being in non-stable periodic orbits. Such trajectories appear chaotic, but they are not.

The parameter values that lead us to the Lorenz Attractor have trajectories that display several properties of turbulence. First, they are non-periodic. In fact, they never intersect themselves when plotted in three dimensions. If they did, then they would start over again with the same initial condition, and the trajectory would be periodic. They also never approach periodic or stationary behaviour.

However, what is particularly interesting about these plots is that their general (rough) geometric form is independent of initial conditions, while the exact form (details) displays sensitive dependence on initial conditions.

One last interesting aspect of the Lorenz Attractor is that its associated trajectories are deterministic. This means that given an exact set of initial conditions, there is only one possible trajectory. Thus, if you were to use exactly the same initial conditions several times, you would be able to reproduce your results.

## CONCLUSIONS

Chaotic phenomena occur not only in meteorology, but also in other fields like biology, demography, psychology. Nevertheless, there is growing evidence that spontaneous, deterministic chaotic dynamics is an important element in understanding the world we live in.

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