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CORIOLIS FORCE AND THE DURABILITY OF INFRASTRUCTURES

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Abstract

The paper deals with the phenomena developed when along the surface of Earth to the well known Coriolis force some reactive forces are opposed. Since, usually at that level, the Coriolis force is a weak one only long lasting interactions become of practical interests. Regarded from this perspective the Coriolis force is first mechanically defined. Its actions in horizontal and vertical planes are further detailed. The conversion law of energy from an inertial system to a non-inertial one is also presented. As a tribute to History of Science the existence of Coriolis force is proved by the Foucault pendulum. Reactive forces to the Coriolis one are usually developed by engineering works like the protection walls of river banks. For such works the Coriolis force acts as a long lasting disturbing action. Like gravity it is a permanent action, but horizontally directed. During the long service of the works, due to imminent soil deformations some structural damages could occur. According to the clause 2.5 of Eurocode 1 the durability of engineering works should be evaluated. The Coriolis force is a determinist entity while the durability is a probabilistic one. As it was already shown in our paper [6] in 2011, this requirement can be reached with the aid of the Mathematical Theory of Reliability only. Considering several computing schemes for reactive forces, the paper is suggesting a range of numerical values for the risk factor. By including in durability analysis the influence of Coriolis force the results are more reliable and faithful to reality.

Key words: *gravity, inertia, reliability, risk factor, safety.*

INTRODUCTION

World population, estimated at 7,141,000,000 people at the beginning of the year 2014, is non-uniformly distributed on the surface of Earth. Most of population is concentrated in mega-cities, while the rest of it is spread in surrounding satellite settlements. The mega-cities cannot live isolated, only by themselves. For surviving they should remain strongly connected between them by complex engineering works, consisting in ways of communication, energetic lines and auxiliary industrial units, generically called critical or vital infrastructures. In spite of the advanced technologies used for providing the safety and durability of these huge investments the number and frequency of occurring some failures remain rather high. Two of the four Aristotelian elements, namely earth and water, are usually involved in these geophysical events. Apparently they seem to be hazards, but in reality they are due to human errors. Nature never makes mistakes. Its laws are sacred (Sofronie,2012).

The paper brings in actuality a mechanical phenomenon well known for its applications in ballistics and meteorological application, namely the Coriolis Effect (Landau, 1965). The historic circumstances in which this effect was discovered and proved are first presented. Then the hypotheses on which the theory of relative motion is based are discussed. The structure of Coriolis force discloses other possible applications of practical interest in engineering works like critical infrastructures.

HISTORICAL DATA

In 1835 the French scientist Gaspard-Gustave de Coriolis (1792-1843) published in Paris the paper *Sur les équations du mouvement relatif des systèmes de corps*. He has shown that the laws of motion could be used in a rotating system of reference if an extra force is added to the motion equations (Laue,1965). In 1836 Coriolis succeeded Louis Marie Henri Navier on the Chair of Applied Mechanics at the École des Ponts and Chaussées in Paris and also to Navier's place in the Académie de Sciences.

His name is one of the 72 names inscribed on the Eiffel Tower.
In 1851, after only 16 years from Coriolis' paper, the French physicist Léon Foucault (1819-1868) provided an experimental demonstration of the Earth rotation around its axis in diurnal motion. For this purpose Foucault used a pendulum, composed by a canon ball with a mass m=26 kg suspended by a cable with a length l=67m from the roof of the Panthéon in Paris. Foucault wrote in his paper that the time in hours taken for the pendulum to return to its original position depends by the latitude at which the experiment is carried out. So, at the poles it takes 24 hours to return to its original position while at the equator it does not rotate at all. Since that date of 3rd February 1851 in Paris Coriolis' Effect was unanimously world widely recognised.
History of Science is mentioning that in time *The Principle of Relativity* assumed successively three versions. The first one is attributed to Galileo Galilee (1564-1642) who defined the inertial systems of references. Galileo assumed that space is homogeneous and isotropic, time is uniform and reversible while the motion is rectilinear and uniform or at rest. In his version relativity principle states that in the inertial systems of reference all laws of mechanics are the identical, and the same are the proprieties of space and time (Landau, 1966). The second version is attributed to Isaac Newton (1642-1727). From the very beginning Newton introduced as model of analysis the material point. It was defined as a body without dimensions, but with mass. In this way Newton has eliminate the three degrees of freedom due to rotation and reduced the whole motion to translation only. All his three laws of motion are invariants with respect to the inertial or Galilean systems of reference (Voinea, 1989). Newton postulated that the existence of an absolute and immobile space cannot be physically proved. Therefore his version of relative principle states that all inertial systems of reference are equivalent, and the laws of mechanics remain invariants with respect of any of them (Voinea, 1989). In 1905 Albert Einstein (1879-1955) lived his *Annus Mirabilis,* i.e. *a miracle year,* and published *the Special Theory of Relativity* that includes all physical phenomena not only the mechanical ones. His theory states that a preferential system of reference with inertial properties does not exist, or at least was not experimentally proved yet. That means a mobile trihedral is defined as inertial or Galilean when translates uniformly and constantly, therefore only when its initial acceleration of translation together with its velocity and acceleration of rotation are zero. All physical laws are invariants with respect of any inertial system of reference. Stephen Hawking calls Galileo Galilee, Isaac Newton and Albert Einstein the Giants of Science [2].

COMPLEMENTARY FORCES AT EARTH SURFACE

Lev Landau (1908-1968), Nobel prized for Physics in 1962, gave an elegant description of the relative motion of a material point into an external field at the surface of the Earth (Landau, 1966). When the motion of the material point is first reported to an inertial or Galilean system of reference K_o in which the velocity of material point is written by v_o and its potential energy with U, the Lagrange function assumes the expression

$$L_o = \frac{mv_o^2}{2} - U \tag{1}$$

that leads to the equation of motion

$$m\frac{d\vec{v}_o}{dt} = -\frac{\partial U}{\partial \vec{r}} \tag{2}$$

For this motion are also successively defined the impulse

$$\vec{p}_o = \frac{\partial L_o}{\partial \vec{v}} = m\vec{v}_o \tag{3}$$

the kinetic moment

$$\vec{M}_o = \vec{r} x \vec{p}_o \tag{4}$$

and the energy

$$E_o = p_o v_o - L_o = \frac{mv_o^2}{2} + U \tag{5}$$

When the same motion is reported to a non-inertial system of reference K that rotates with a constant and uniform angular velocity, measured in rad/s,

$$\vec{\Omega} = const. \tag{6}$$

then the recurrence relation between the velocities in the two systems of reference K_o and K, with the same position vector r, does exist

$$\vec{v}_o = \vec{v} + \vec{\Omega}x\vec{r} \tag{7}$$

In these conditions the above defined functions become as follows:
Lagrange function

$$L = \frac{mv^2}{2} + m\vec{v}(\vec{\Omega}x\vec{r}) + \frac{m}{2}(\vec{\Omega}x\vec{r})^2 - U \tag{8}$$

Equation of motion

$$m\frac{d\vec{v}}{dt} = -\frac{\partial U}{\partial \vec{r}} + 2m(\vec{v}x\vec{\Omega}) + m\vec{\Omega}x(\vec{r}x\vec{\Omega}) \tag{9}$$

By comparing the two equations of the motion for inertial and non-inertial systems, (2) and (9) respectively, it is noticed that in the second equation there are two additional terms. Both have the dimension of force, due to the mass m they have an inertial nature and since Ω is the angular velocity of Earth rotation they have a permanent character. The first additional term with the expression

$$\vec{F}_C = 2m(\vec{v}x\vec{\Omega}) \tag{10}$$

was named the complementary force of Coriolis. Since this amount is not generated by an interaction, according to Newtonian definition of force, it is not a proper force, but as long as it is able to cause motion is accepted as such. The second additional term with the expression

$$\vec{F}_{cf} = m\vec{\Omega}x(\vec{r}x\vec{\Omega}) \tag{11}$$

is the complementary centrifugal force. Since the numerical value of Earth angular velocity

$$\Omega = \frac{2\pi}{24}\frac{rad}{hour} = \frac{2\pi}{24x60x60}\frac{rad}{s} =$$
$$= 0.727x10^{-4}\frac{rad}{s}$$

is rather small, its quadratic value becomes much smaller. Generally, since

$$F_{cf} << F_C$$

the centrifugal force could neglected when is compared with the Coriolis one.
Further, the impulse in the non-inertial reference system

$$\vec{p} = \frac{\partial L}{\partial \vec{v}} = m\vec{v} + m\vec{\Omega}x\vec{r} = m(\vec{v} + \vec{\Omega}x\vec{r}) = \\ = m\vec{v}_o = \vec{p}_o \tag{12}$$

coincides with the impulse in the inertial system.
Kinetic moment in the non-inertial reference system

$$\vec{M} = \vec{r}x\vec{p} = \vec{r}x\vec{p}_o = \vec{M}_o \tag{13}$$

also coincides with the kinetic moment in the inertial system.
Finally, the energy

$$E = \vec{p}\vec{v} - L = \frac{mv^2}{2} - \frac{m}{2}(\vec{\Omega}x\vec{r})^2 + U \tag{14}$$

where the second term is an additional potential energy called centrifugal energy. Replacing the velocity with its value from (7) one obtains

$$E = \frac{mv_o^2}{2} + U - m(\vec{r}x\vec{v}_o)\vec{\Omega} \tag{15}$$

or shortly

$$E = E_o - \vec{M}_o\vec{\Omega} \tag{16}$$

This last equation represents the law of transformation the energy of material point when one passes from an inertial system of reference to a non-inertial system of reference.

CORIOLIS FORCE

For practical purposes the above vector equations are replaced with their analytical forms. One assumes that the material point is located in the northern hemisphere at the latitude λ, and the origin of reference system O is located in the centre of the Earth. The axis Oz is vertically oriented, the axis Oy downwards, in meridian direction, and axis Ox is horizontally oriented, perpendicularly on the vertical plan yOz (Fig.1).

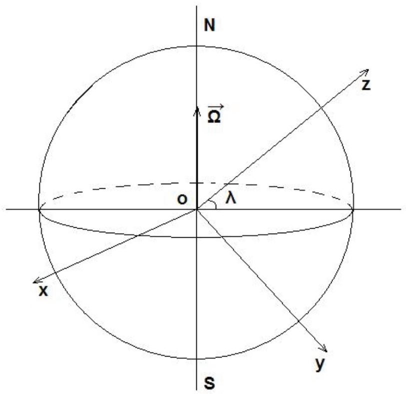

Figure 1. Reference axes

For the three vectors in the expression of Coriolis force

$$\vec{F}_C = -2m(\vec{\Omega} x \vec{v}) \tag{17}$$

with the following notations $\vec{F}_C(F_{Cx}, F_{Cy}, F_{C_z})$, $\vec{\Omega}(\Omega_x, \Omega_y, \Omega_z)$ and $\vec{v}(v_x, v_y, v_z)$ one obtains

$$\vec{F}_C = (\Omega_y v_z - \Omega_z v_y)\vec{i} + (\Omega_z v_x - \Omega_x v_z)\vec{j} + \\ +(\Omega_x v_y - \Omega_y v_x)\vec{k} \tag{18}$$

Since the angular velocity decompose in the vertical plan yOz $\vec{\Omega}(0, -\Omega\cos\lambda, \Omega\sin\lambda)$ the expression (17) assumes the form

$$\vec{F}_C = 2m\Omega(v_z \cos\lambda + v_y \sin\lambda)\vec{i} - \\ -2m\Omega(v_x \sin\lambda)\vec{j} - 2m\Omega(v_x \cos\lambda)\vec{k} \tag{19}$$

In the case of a material point that falls vertically $\vec{v}(0,0,-v)$ the expression (19) becomes

$$\vec{F}_C = -(2m\Omega v \cos\lambda)\vec{i} \tag{20}$$

and its unique horizontal component is

$$F_{Cx} = -2m\Omega v \cos\lambda \tag{21}$$

Due to its minus sign the falling material point is horizontally deviated towards east. The highest deviation occurs at equator where the latitude is $\lambda = 0$,

$$F_{Cx}^{\max} = -2m\Omega v \tag{22}$$

and any deviation does not occur at all at pole where $\lambda = 90°$.

In the case of a material point that moves on a horizontal plan in a direction deviated from the axis Ox with the angle α. In these conditions the expression (19) becomes

$$\vec{F}_C = 2m\Omega v(\sin\lambda \sin\alpha)\vec{i} - \\ -2m\Omega v(\sin\lambda \cos\alpha)\vec{j} \tag{23}$$

where its components are

$$F_{Cx} = 2m\Omega v \sin\lambda \sin\alpha \tag{24}$$

and

$$F_{Cy} = -2m\Omega v \sin\lambda \cos\alpha \tag{25}$$

Since

$$\vec{F}_C \vec{v} = F_{Cx} v_x + F_{Cy} v_y = \\ = 2m\Omega v^2 (\sin\lambda \sin\alpha \cos\alpha - \sin\lambda \cos\alpha \sin\alpha) = 0$$

means that $\vec{F} \perp \vec{v}$ and the complementary force Coriolis deviates the motion of material point perpendicular on the direction of velocity $\vec{v}$ and on its right side. At equator, where $\lambda = 0$, $F_C = 0$, and Coriolis effect does not exist while at pole, where $\lambda = 90°$, and for $\alpha = 0$,

$$F_{Cy}^{\max} = -2m\Omega v \tag{26}$$

With the aid of above determined expressions of the Coriolis force it was found out that the period in which the vertical plan of Foucault pendulum completely rotates assumes the simple expression

$$T = \frac{24 hours}{\sin\lambda} \tag{27}$$

and for $\lambda = 45°$, $T \cong 34 hours$. This period does not depend either by the length or mass of the pendulum used for demonstration.

DURABILITY OF CRITICAL INFRASTRUCTURES

The Coriolis force is as permanent as the gravitational force, but comparatively its amount is much smaller. That means both forces are long lasting and never cease their actions. Like gravitational force the Coriolis one is a stationary force because it does not explicitly depends on time, but however it essentially differs by gravity; Coriolis force is a circular one because it depends by the velocity of motion and non-conservative because its mechanic work depends on the trajectory of motion. In addition, if the gravitational force is a central one with a unique direction and sense, as that of Earth centre, the Coriolis force follows any trajectory in space, namely that decided by the vector product between the velocity of mobile and Earth's rotation. When the two forces, gravitational and Coriolis, are acting together on the same point usually the last one is neglected as non-significant. On the contrary, when Coriolis force does not feel the influence of gravitational force its action could become significant for the mechanical state of material points. In this particular situation, in engineering applications at least, the complementary Coriolis force has a perception of a disturbing one.

The existence of critical infrastructures, as fixed and not movable constructions, is governed by the laws of equilibrium based on Newton's Third Principle of reciprocal actions. Casual and unavoidable deformations or displacements are restricted by Jacob Bernoulli's hypothesis such as their mechanical state to be the rest. For analysis purpose, according to the existing codes in force, the Eurocode 1 including, two limit states are assumed: 1) The ultimate limit state what is applied for safety analysis and 2) The limit state of service what is applied for durability analysis. Durability is a probabilistic concept based on the Mathematic Theory of Reliability (Sofronie, 2012). In a general form the durability function is defined as

$$\tau(t) = \int_0^\infty F(t)dt \tag{28}$$

where the reliability function

$$F(t) = e^{-\int_o^t \lambda(t)dt} \tag{29}$$

dramatically decreases with the function of risk $\lambda(t)$. In the most simplified case the durability and risk factor follow a typical relation of inverse proportionality

$$\tau = \frac{1}{\lambda} \tag{30}$$

Risk factors and risk function as well are evaluated with the methods of Statistical Mathematics. The results obtained are classified on different scales from the smallest to highest risks. Statistical analysis also includes the construction degrees of importance. Obviously, the critical infrastructures being vital ones are credited with the highest degree of importance and consequently even the smallest risks are taken into account.

Critical infrastructures were not discovered by the modern technologic civilization. They were well known by all ancient societies like City Wall in Babylon, Chinese Wall in Far Orient and the networks of roads, bridges and aqueducts built during the Roman Empire, for instance. Only the name of *critical infrastructures* was recently invented. Such a name was necessary to define the massive engineering works, world widely built now.

The risks that could occur in critical infrastructures due to the longtime action of the horizontal components of Coriolis force are coming from the rheological phenomena developed in earthen works and geosynthetic materials for which the elasticity law of Robert Hooke does no longer subsists. The simplest form of the rheological state is expressed by the equation

$$\sigma + n\frac{d\sigma}{dt} = E\varepsilon + m\frac{d\varepsilon}{dt} \tag{31}$$

where *m* and *n* are dimensional parameters.

For *n=0* one obtains the general law of creep phenomenon

$$\varepsilon = \frac{\sigma}{E}(1 - e^{-\frac{E}{m}t}) \tag{32}$$

or for $\sigma = const.$ and $m/n = E_1$

$$\varepsilon = \frac{\sigma}{E} + \sigma(\frac{1}{E_1} - \frac{1}{E})e^{-\frac{E}{m}t} \tag{33}$$

while for *m=0,* the law of stress relaxation phenomenon due to Maxwell

$$\sigma = \sigma_o e^{-\frac{t}{m}} \tag{34}$$

Both phenomena of creep and stress relaxation are latent and slow, but fortunately they are warning by cracks or visible superficial strains.

If by periodical inspections the danger is detected in due time great damages and even fatalities could be avoided.

CONCLUSIONS

Although the theory of Coriolis force is well known for long time it was less applied in engineering works. That is probably because from very beginning it was called a complementary force. The paper emphasized the important role of this force in the maintenance of critical infrastructures. With the existing computing facilities the Coriolis force can be easily included in all engineering analyses. The theoretical base presented in the paper is helpful in correct interpretation on any analysis results.

REFERENCES

Buchholtz, N.N. et al., 1952. Culegere de probleme de Mecanică rațională. Editura Tehnică, București, p.210-216.
Hawking, Stephen W. (Ed.), 2003. *On the Shoulders of Giants.* Penguin Books, London, p.391-1167.
Landau, L.D., Kitaigorodski, A.I., 1965. Fizica pentru toți. Editura Științifică, București, p.64-70.
Landau, L.D., Lifșiț, E.M., 1966. Mecanica. Editura Tehnică, București, p.176-182.
Laue, Max von, 1965. Istoria fizicii, Editura Științifică, București. p.25-41.
Pascu R., Sofronie R., 2011. Durability of critical infrastructures. AES Bioflux 3(2):167-186.
Sofronie, R., 2012. Agriculture and euphemistic view on physical hazards. Proceedings of the International Conference "AGRICULTURE FOR LIFE, LIFE FOR AGRICULTURE" Vol.12, Issue 4, 2012 Bucharest.
Strelkov, S., 1978. Mécanique. Édition Mir, Moscou.
Voinea, R. et al., 1989. Introducere în mecanica solidului cu aplicații în inginerie. Editura Academiei, București, p.229-232.