

5 METHODS OF DETERMINING THE CHARACTERISTIC VALUES OF SHEAR STRENGTH PARAMETERS

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Abstract

The shear strength parameters, internal friction angle (ϕ [°]) and cohesion (c [kPa]) represent derived values, according to the methodology of Eurocode 7 and NP 122. They are determined by processing the pairs of normal stress (σ) - tangential stress (τ) values resulted from direct shear or triaxial compression tests. The paper presents 5 methods of determining the characteristic values, following the direct processing of ϕ and c values or indirectly, by processing pairs of σ - τ values, resulted from direct shear tests. The characteristics values of shear strength parameters are required for the geotechnical design in the case of various geohazards and for foundation solution. A series of conclusions are drawn based on the legislation in force.

Key words: characteristic values, cohesion, derived values, internal friction angle, shearing strength parameters.

INTRODUCTION

From a geotechnical engineering perspective, there are two types of parameters which are essential in analysing the mechanical behaviour of soils: the parameters governing the relation between stresses and strains (compressibility and consolidation parameters) and the parameters governing the development of plastic strains which lead to failure.

Prästings et al. (2019) said that the lack of harmonisation between reliability-based design and the partial factor method in Eurocode 7 is preventing the widespread introduction of a risk-based concept in geotechnical design.

The current paper deals with the determination of the values to be used for design, namely the inferior characteristic values of the shearing resistance parameters, as used in Mohr-Coulomb constitutive model.

The choosing of the characteristic values is anything but trivial, due to the highly heterogeneous nature of soils and the variation of the parameters with respect to the method used for analysing the shearing resistance, in particular.

Another particular property of soils that increase the difficulty in choosing the

parameters is the dependence on the state of stress in which the soil is at a particular moment and more than that, on the history of the loading states.

The current paper analyses the fit of usage for five methods of computing the characteristic values of the shearing resistance parameters, by analysing the current technical norms in use and prior methods prescribed by the legislation in Romania. For that, several direct shearing tests have been employed, analysing the results from the point of view of the safety the five methods provide.

The methods used in this paper are based only on the stochastic approach, without using any other numerical methods for analysing the fitness of the methods.

MATERIALS AND METHODS

Using following notations:

X_i - measured values, with $i \in [0, n]$;

X_m - average of the measured values X_i ;

s_x - standard deviation of the measured values

X_i ;

V_x - coefficient of variation;

X_k - characteristic value (there will be a superior value and an inferior value);

t_α - statistical coefficient derived from STAS 3300/1-1985.

Computation procedures considered are:

- According to NP 122-2010

The average of the measured values is computed using equation (1):

$$X_m = \frac{\sum_{i=0}^n X_i}{n} \quad (1)$$

The standard deviation of the measured values is computed using equation (2):

$$s_x = \sqrt{\frac{\sum_{i=0}^n (X_i - X_m)^2}{n - 1}} \quad (2)$$

The coefficient of variation is computed using equation (3)

$$V_x = \frac{s_x}{X_m} \quad (3)$$

The superior and inferior characteristic values are computed using equation (4). The statistical coefficient k_n is determined using Table 3.2 from NP 122-2010.

$$X_{k_sup} = X_m(1 + k_n V_x) \quad (4)$$

$$X_{k_inf} = X_m(1 - k_n V_x)$$

The previous computation procedure was applied in two different hypotheses. The first hypothesis determined the values of the shear resistance parameters for each of the direct shearing tests performed and applied the previously described procedure to compute the characteristic values of the shear resistance parameters.

The second hypothesis considered the maximum shearing stress determined from the direct shearing tests (the normal stress for each test was imposed). The characteristic values for maximum shearing stresses were determined for each of the imposed normal stresses and the shearing resistance parameters were determined using the pairs of the characteristic shearing stresses and the imposed normal stresses.

- According to NP 122-2010 Appendix A.4

In appendix A.4 of NP 122-2010 certain values are recommended for the coefficient of variation V_x , depending on the analysed parameters. For the current case, the recommended coefficient for the internal friction angle ϕ' , applied on the tangent of the angle is 0.1. Similar, the recommended value for the drained and undrained cohesion is 0.4.

- According to NP 122-2010 Appendix A.5

The procedure uses a regression line, having the equation $\tau = \sigma \tan \phi + c$ and the characteristic values are obtained based on the statistical coefficient t_α .

$$y_k = a_k x + b \pm t_\alpha s_c \quad (5)$$

$$t_\alpha = f(n - 1)$$

As a result, the correlation that gives the characteristic value is parallel to the regression line calculated by the least squares method. Therefore, only the cohesion value is underestimated, and a characteristic value is provided, while the internal friction angle has the same value, from the regression line.

- According to the procedure from STAS 3300/1-1985 Appendix A

Before the entry into force of Eurocode 7 and NP 122, the calculation values of the geotechnical parameters are determined according to STAS 3300-1/85 which contains distinct procedures for independent parameters (ρ , w , E_{oed}) and correlated parameters (ϕ and c).

The procedure described in STAS 3300/1-1985, that can be fully applied because it uses mathematical statistics relationships, is based on the following equations:

$$y_k = a_k x + b_k \quad (6)$$

$$a_k = a_m \left(1 \pm \frac{t_\alpha s_{x,a}}{a_m}\right)$$

$$b_k = b_m \left(1 \pm \frac{t_\alpha s_{x,b}}{b_m}\right)$$

$$t_\alpha = f(n - 2)$$

As a result, both the internal friction angle and the cohesion are underestimated (inferior characteristic values are obtained).

The coefficient of variation, through the recommended maximum values for physical characteristics of soil, underlies the delimitation of geological strata (Olinic, 2014).

RESULTS AND DISCUSSIONS

For obtaining the input data for the current paper, 36 direct shearing tests were performed. The tests were conducted with imposed normal stress (12 tests with an imposed value of 121.37 kPa, 12 tests with an imposed value of 216.75 kPa, and 12 tests with an imposed value of 312.12 kPa). The maximum shearing stress was recorded for all the 36 tests and presented in the Table 1.

Table 1. Measured maximum shearing stresses for the performed direct shearing tests [kPa]

Measured values	σ_1	σ_2	σ_3
		121.37	216.75
	τ_1	τ_2	τ_3
X ₁	132.66	159.12	187.23
X ₂	155.00	194.10	228.12
X ₃	148.92	201.91	242.62
X ₄	130.56	172.90	208.00
X ₅	148.92	166.29	212.50
X ₆	132.22	172.34	223.10
X ₇	141.13	186.67	231.66
X ₈	161.19	199.12	228.12
X ₉	125.00	168.44	207.48
X ₁₀	151.70	177.92	200.23
X ₁₁	112.22	163.42	198.56
X ₁₂	167.88	198.00	243.74
Minimum values, X _{min}	112.22	159.12	187.23
Maximum values, X _{max}	167.88	201.91	243.74
Average values, X _m	142.28	180.02	217.61

The measured results were plotted in an σ - τ coordinates graph, presented in Figure 1.

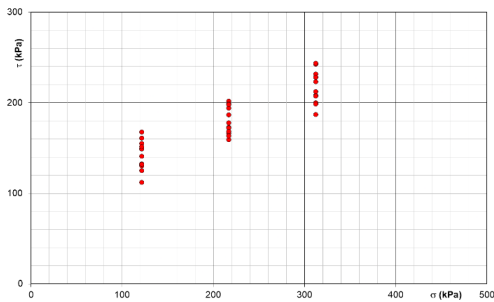


Figure 1. Measured values plotted in σ - τ coordinates.

From the measured stresses, the shearing resistance parameters were derived. The obtained values are presented in Table 2.

Table 2. Derived values of the shearing resistance parameters

Derived values	ϕ_{CU}	$\tan \phi_{CU}$	C _{CU}
	°	-	kPa
X ₁	15.96	0.286	97.67
X ₂	20.96	0.383	109.39
X ₃	26.16	0.491	91.34
X ₄	22.25	0.409	82.01
X ₅	18.44	0.333	110.31
X ₆	25.47	0.476	72.62

X ₇	25.39	0.475	83.63
X ₈	19.34	0.351	120.08
X ₉	23.38	0.432	73.25
X ₁₀	14.27	0.254	121.48
X ₁₁	24.35	0.453	59.96
X ₁₂	21.69	0.398	117.01
Minimum values, X _{min}	14.27	0.254	59.96
Maximum values, X _{max}	26.16	0.491	121.48
Average values, X _m	21.47	0.395	94.90

The obtained values were plotted in a ϕ -c coordinate graph presented in Figure 2.

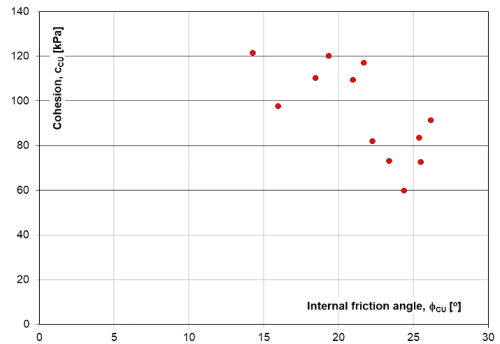


Figure 2. Derived shear resistance values plotted in ϕ -c coordinates.

Following the computation procedure described in the previous section, using the provisions of NP 122-2010, the following characteristic values were derived (Table 3).

Table 3. Derived values of the characteristic shearing resistance parameters

Number of selected X _i values, n	12	12	12
Standard deviation, s _x	3.84	0.0767	20.8
Coefficient of variation, V _x	0.179	0.194	0.220
k _n for V _{x unknown}	0.51	0.51	0.51
X _{k, sup} = X _m (1+k _n V _{x unknown})	23.44	0.434	105.55
X _{k, inf} = X _m (1-k _n V _{x unknown})	19.51	0.356	84.24
$\phi_{CU k, inf}$		19.59	

The derived values were plotted in a σ - τ graph, together with the measured values for stresses, showed in Figure 3.

The same computation procedure was applied for the values of the measured stresses. The characteristic values that were obtained are presented in Table 4.

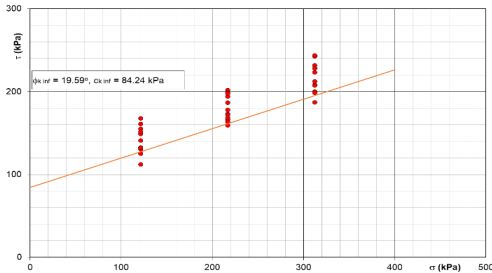


Figure 3. Derived characteristic values for the shearing resistance parameters using NP 122-2010 together with the measured values plotted in σ - τ coordinates.

Table 4. Characteristic values of the shearing stresses using NP 122-2010 and derived characteristic shearing resistance parameters

Number of selected X_i values, n	12	12	12
Standard deviation, s_x	16.20	15.25	18.00
Coefficient of variation, V_x	0.114	0.085	0.083
k_n for $V_{x\text{unknown}}$	0.51	0.51	0.51
$X_{k_sup} = X_m(1+k_n V_{x\text{unknown}})$	150.57	187.82	226.82
$X_{k_inf} = X_m(1-k_n V_{x\text{unknown}})$	134.00	172.22	208.40
$\phi_{CU\ k\ inf}$		21.31	
$c_{CU\ k\ inf}$		87.00	

The derived characteristic values of the shearing resistance parameters were plotted in a σ - τ graph, together with the measured stresses (Figure 4).

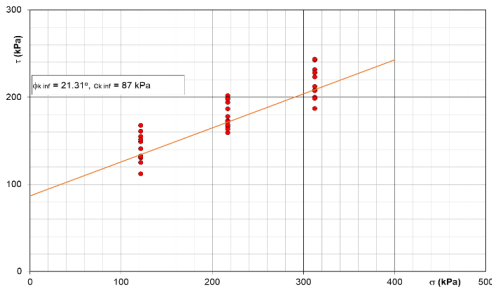


Figure 4. Derived characteristic values for the shearing resistance parameters from the characteristic derived stresses using NP 122-2010 together with the measured values plotted in σ - τ coordinates

Using the computation procedure described in NP 122-2010 Appendix A.4, the coefficients of

variation V_x for the $\tan(\phi)$ and c were considered 0.1 and 0.4 respectively. With these values, the derived characteristic values for the shearing resistance parameters were computed, using the values presented in Table 2. The results are presented in Table 5.

Table 5. Derived characteristic values of the shearing stresses using NP 122-2010 Appendix A.4

Number of selected X_i values, n	12	12	12
Standard deviation, s_x	3.84	0.0767	20.8
Coefficient of variation, V_x	0.179	0.100	0.400
k_n for $V_{x\text{unknown}}$	0.51	0.51	0.51
$X_{k_sup} = X_m(1+k_n V_{x\text{unknown}})$	23.44	0.415	114.31
$X_{k_inf} = X_m(1-k_n V_{x\text{unknown}})$	19.51	0.375	75.48
$\phi_{CU\ k\ inf}$		20.55	

The characteristic shearing resistance parameters obtained using the appendix A.4 were plotted in a σ - τ graph, together with the measured stresses (Figure 5).

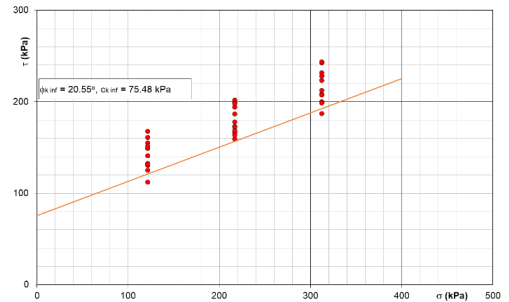


Figure 5. Derived characteristic values for the shearing resistance parameters using NP 122-2010 Appendix A.4 together with the measured values plotted in σ - τ coordinates

Following the computation procedure described in NP 122-2010 A.5, the trendline was derived from the measured stresses. The trendlines and values obtained are presented in Figure 6.

Applying the procedure described in STAS 3300/1 - 1985 to improve the results obtained from NP 122-2010 Appendix A.5, new trendlines are obtained. The results are presented in Figure 7.

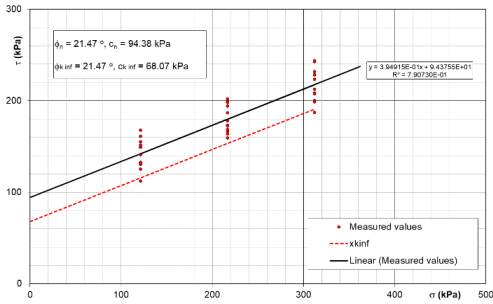


Figure 6. Derived characteristic values for the shearing resistance parameters using NP 122-2010 Appendix A.5 together with the measured values plotted in σ - τ coordinates

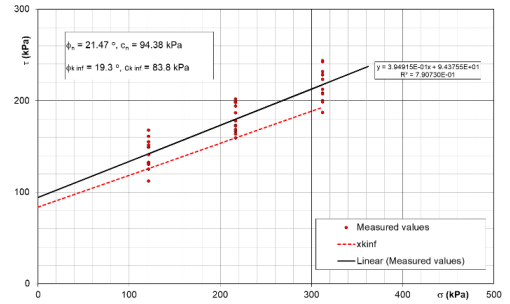


Figure 7. Derived characteristic values for the shearing resistance parameters using NP 122-2010 Appendix A.5, improved with the procedure from STAS 3300/1-1985 together with the measured values plotted in σ - τ coordinates

Table 6. The characteristic inferior values of the shearing resistance parameters obtained through the five methods described in the current paper

Characteristic inferior values of the shearing resistance parameters	NP 122-2010 - derived values – Method 1	NP 122-2010 - inferior values for stresses - Method 2	NP 122-2010 - App A.4 - Method 3	NP 122-2010 App A.5 - Method 4	NP 122-2010 Appendix A.5+STAS 3300/1 -1985 - Method 5
$\phi_{CU\ k\ inf}$	19.6 °	21.3 °	20.6°	21.5 °	19.3 °
$c_{CU\ k\ inf}$	84.2 kPa	87.0 kPa	75.5 kPa	68.1 kPa	83.8 kPa

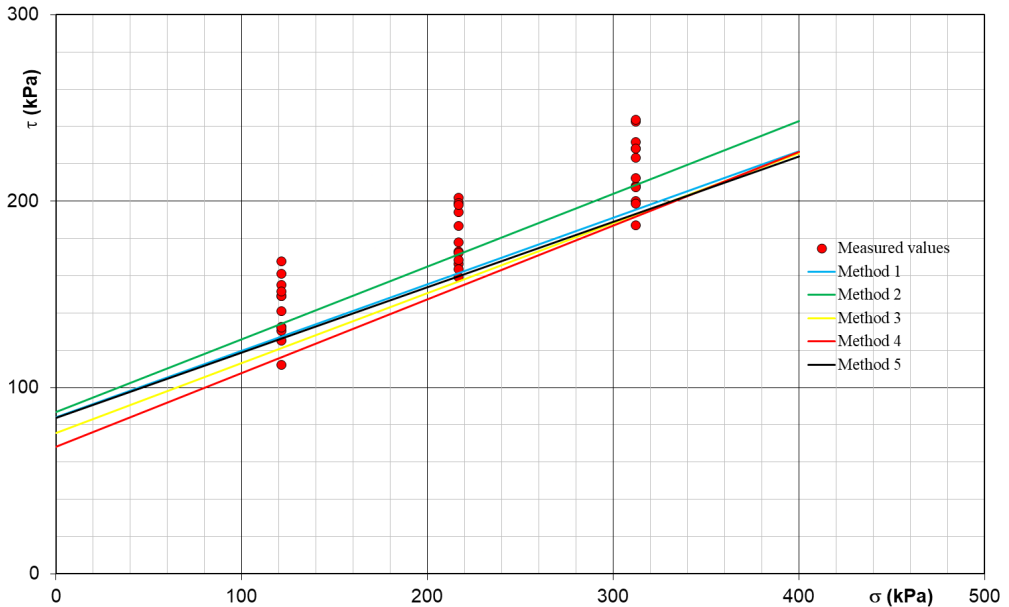


Figure 8. Mohr-Coulomb failure lines plotted using the inferior characteristic values of the shearing resistance parameters determined using the five methods described in the paper

CONCLUSIONS

The current paper analyses the use of five different methods of computing the inferior characteristic values of the shearing resistance

parameters (Table 6), using the Mohr-Coulomb constitutive model formulation. If considering the results plotted in Figure 8, the first conclusion is that Method 2 doesn't provide sufficient reliability to be used.

On the other hand, considering that Mohr-Coulomb constitutive model is highly dependent on the normal stress applied, and considering that all the other methods (except for Method 2, of course) provide results that are inferior to most of the measured stresses – smaller than 95% of the obtained results, the choice of the method used relies rather on the designer.

The recommendation of the authors is to choose the usage of Method 4 for the situations where the normal stresses are low enough (up to 200 kPa) and Method 5 for normal stresses higher than 200-250 kPa.

Methods 1, 3 and 4 are prescribed by the current design norms and choosing any of them will be in accordance with the laws in use. As one can observe in Figure 8, all of them provide sufficient reliability.

More tests on more types of materials can provide with a better understanding of the methods for determining the characteristic values, considering that different types of materials might provide improved results.

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